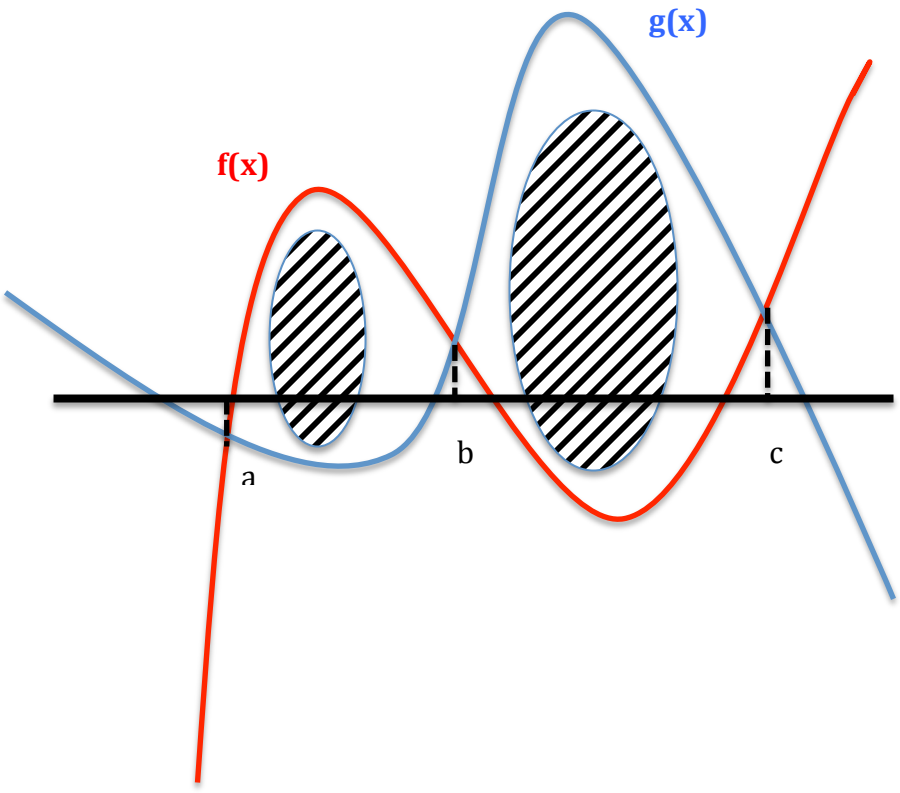
Carleton UNIVERSITY
Canada's Capital University

Mathematics Tutorial Series
Integral Calculus #19

Integrals and Area - 2

Sometimes we want the area of a region bounded by two curves.

1. Find the intersections.
2. Work out which function is on top.
3. Use a new integral for each interval.



Area =

$$\int_a^b TOP - BOTTOM \; dx$$

Here is a question from a recent first year examination.

Calculate the area bounded by $f(x) = x^2 - 4$ and $g(x) = -x^2 + 4$.

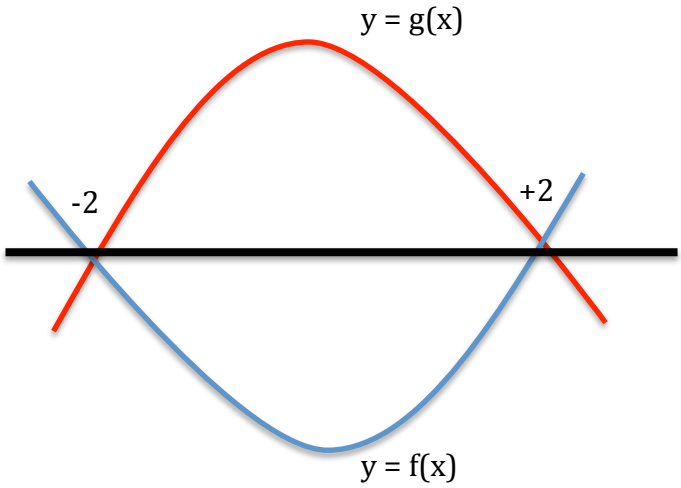
These are quadratic polynomials so they graph as parabolas – one opening up and the other opening down.

1. Where do they intersect?

Let $f(x) = g(x)$ so $x^2 - 4 = -x^2 + 4$.

Then $2x^2 = 8, x^2 = 4$ and $x = \pm 2$.

2. Which is on top?



Area =

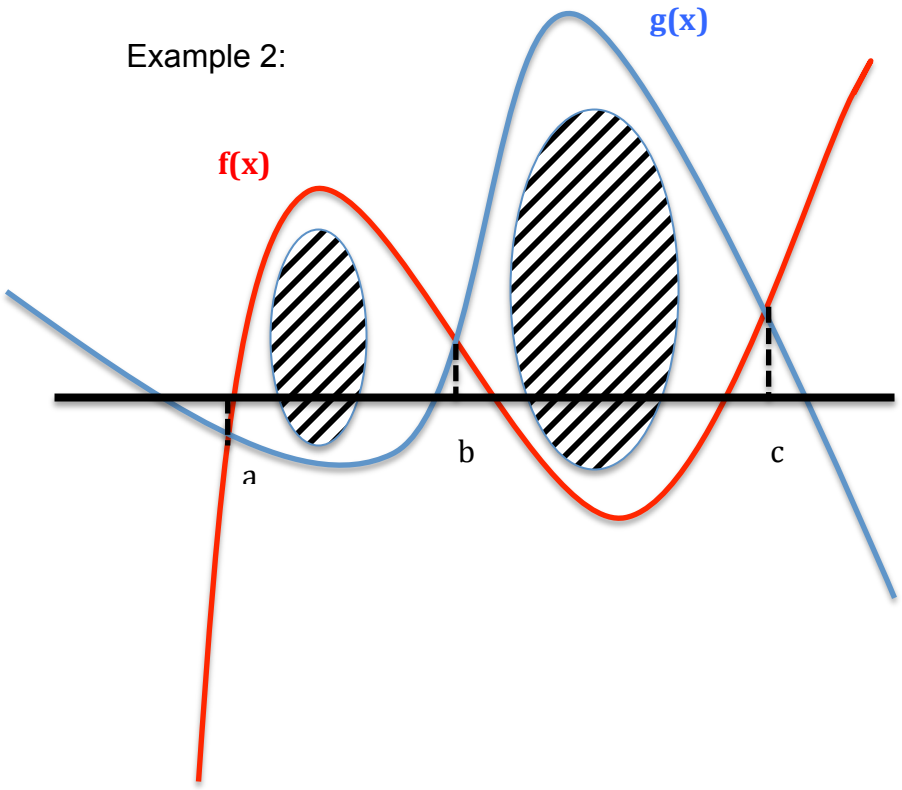
$$\int_a^b TOP - BOTTOM \, dx$$

$$\int_{-2}^{+2} (-x^2 + 4) - (x^2 - 4) \, dx$$

$$= \int_{-2}^{+2} 8 - 2x^2 \, dx = 8x - \frac{2}{3}x^3 \Big]_{-2}^{+2}$$

$$= 16 - \frac{2}{3}8 - (-16) + \left(-\frac{2}{3}8\right) = \frac{2}{3}(32) = \frac{64}{3}$$

Example 2:



Area =

$$\int_a^b f(x) - g(x)\ dx + \int_b^c g(x) - f(x)\ dx$$

Area =

$$\int_a^b TOP - BOTTOM\ dx$$